Multi-channel Wireless Networks with Infrastructure Support: Capacity and Delay

Hong-Ning Dai* Raymond Chi-Wing Wong† and Qinglin Zhao*
*Macau University of Science and Technology, Macau
hndai@ieee.org zqlict@hotmail.com
†Hong Kong University of Science and Technology, Hong Kong
raywong@cse.ust.hk

***Abstract*—In this paper, we propose a novel multi-channel wireless network with infrastructure support, called an *MC-IS* network. To the best of our knowledge, we are the first to study the capacity and the delay of such an *MC-IS* network. In particular, we derive the upper bounds and the lower bounds on the network capacity of such *MC-IS* networks contributed by ad hoc communications, where the orders of the upper bounds are the same as the orders of the lower bounds, implying that the bounds are tight. We also found that the capacity of *MC-IS* networks contributed by ad hoc communications is mainly limited by *connectivity* requirement, *interference* requirement, *destination-bottleneck* requirement and *interface-bottleneck* requirement. In addition, we also derive the average delay of *MC-IS* networks contributed by ad hoc communications, which is bounded by the maximum number of hops.**

I. Introduction

In this paper, we propose a novel multi-channel network with infrastructure support, which is called an *MC-IS* network. An *MC-IS* network consists of *common nodes* (or nodes), each with a single network interface card (NIC), and *infrastructure nodes* (or base stations), each with multiple NICs, where infrastructure nodes are connected via a *wired* network that has much higher bandwidth than a wireless network of common nodes. Both common nodes and infrastructure nodes can operate on different channels. Besides, an *MC-IS* network has the following additional characteristics.

- Each node with a single NIC can communicate with either another node or a base station. But, a node supports only one transmission or one reception at a time. Besides, it cannot simultaneously transmit and receive (i.e., it is in a *half-duplex* mode).
- Each base station with multiple NICs can communicate with more than one node. In addition, a base station can work in a *full-duplex* mode, i.e., transmissions and receptions can occur in parallel.

Take Fig. 1 as an example of *MC-IS* networks. In this network, n nodes are randomly, uniformly and independently distributed on a unit square plane A. Each node is mounted with a single interface that can switch to one of C available channels and it can be a data source or a destination. All the nodes have the same transmission range. Besides, there are b *base stations*, where b is assume to be a square of an integer b_0 (i.e., $b = b_0^2$). Each base station is equipped with m interfaces, each of which is associated with a single interface

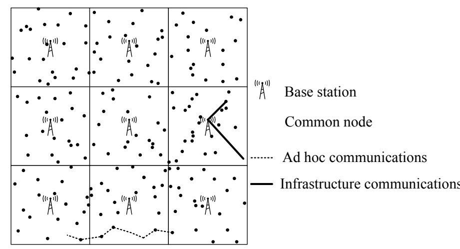

Fig. 1. Network topology of an *MC-IS* network

that can operate on one of C channels. The plane A is evenly partitioned into b equal-sized squares, which are called *BS-cells*. We assume that a base station is placed at the center of each *BS-cell*. Unlike a node, a base station is neither a data source nor a destination and it only helps forwarding data for nodes. All the base stations are connected through a wired network without *capacity constraint* and *delay constraint*.

There are two kinds of communications in an *MC-IS* network: (i) *Ad hoc communications* between two nodes, which often proceed in a multi-hop manner; (ii) *Infrastructure communications* between a node and a base station, which span a single hop, as shown in Fig. 1. An infrastructure communication consists of an *uplink* infrastructure communication from a node to a base station, and a *downlink* infrastructure communication from a base station to a node.

In this paper, we consider the H*-max-hop routing strategy*, in which, if the destination is located within H ($H \geq 1$) hops from the source node, data packets are transmitted in ad hoc communications. Otherwise, data packets are forwarded in infrastructure communications. The base station then relays the packets through the wired network. After the packets arrive at the base station that is closest to the destination node, the base station then forwards the packets to the destination node (i.e., the downlink infrastructure communication). It is obvious that *when there is an uplink communication, there is always a downlink communication*. We then divide the total bandwidth of W bits/sec into three parts: (1) W_A for ad hoc communications, (2) $W_{I,U}$ for uplink infrastructure communications and (3) $W_{I,D}$ for downlink infrastructure communications. Since $W_{I,U}$ is equal to $W_{I,D}$, it is obvious that $W = W_A + W_{I,U} + W_{I,D} = W_A + 2W_{I,U}$. To simplify our analysis, we use W_I to denote either $W_{I,U}$ or $W_{I,D}$.

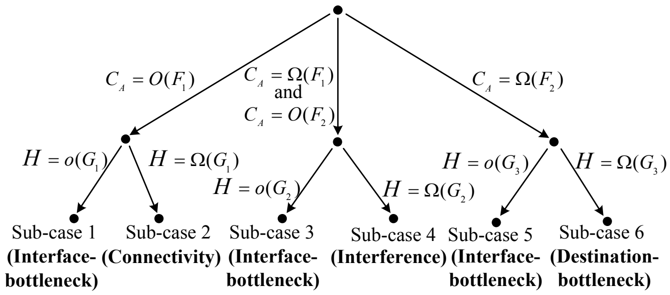

Fig. 2. All possible sub-cases considered

TABLE I
THE MAIN RESULTS

Conditions	Per-node Throughput λ	Delay D
Connectivity Condition	$\Theta(\frac{W_A}{HC_A \log n})$	$\Theta(H)$
Interference Condition	$\Theta(\frac{W_A}{C_A^{\frac{1}{2}} H \log^{\frac{1}{2}} n})$	
Destination-bottleneck Condition	$\Theta(\frac{n^{\frac{1}{2}} \log\log(H^2 \log n) W_A}{C_A H \log^{\frac{1}{2}} n \cdot \log(H^2 \log n)})$	
Interface-bottleneck Condition	$\Theta(H^2 \frac{\log n}{n} \cdot \frac{W_A}{C_A})$	

Corresponding to the partition of the bandwidth, we also split the C channels into two disjoint groups C_A and C_I, in which C_A channels are dedicated for ad hoc communications and C_I channels are dedicated for infrastructure communications. Thus, $C = C_A + C_I$. Besides, each base station is mounted with m NICs, which serve for both the uplink traffic and the downlink traffic. It is obvious that the number of NICs serving for the uplink traffic is equal to the number of NICs serving for the downlink traffic. So, m must be an even number.

To the best of our knowledge, we are the first to propose such an *MC-IS* network, which has not been studied in the literature.

A. Contributions and Main Results

The primary research contributions of our paper are summarized as follows.

(1) We formally identify an *MC-IS* network that characterizes the features of *multi-channel* wireless networks with *infrastructure support*. The capacity and the average delay of an *MC-IS* network have not been studied before.

(2) We derive both the *upper bounds* and the constructive *lower bounds* of the *capacity* of an *MC-IS* network contributed by ad hoc communications. Importantly, the orders of the lower bounds are the same as the orders of the upper bounds, meaning that the upper bounds are tight. We also derive the delay of an *MC-IS* network contributed by ad hoc communications.

(3) We found that the capacity of an *MC-IS* network contributed by ad hoc communications is mainly limited by four requirements - *connectivity requirement*, *interference requirement*, *destination-bottleneck requirement* and *interface-bottleneck requirement*.

Regarding (1), we identify the characteristics of an *MC-IS* network and describe the network topology, the network communications and the routing strategy in Section II, which also presents the models and assumptions used in this paper.

We then derive the upper bounds on the network capacity contributed by ad hoc communications in Section III and the constructive lower bounds in Section IV, both of which bring us to (2) above.

With regard to (3), we summarize our main results in Table I, which are stated in Theorem 1 and Theorem 2. Specifically, we found that the capacity of an *MC-IS* network is mainly limited by four requirements: (i) *Connectivity requirement* - the need to ensure that the network is connected so that each source node can successfully communicate with its destination node; (ii) *Interference requirement* - two receivers simultaneously receiving packets from two different transmitters must be separated with a minimum distance to avoid the interference; (iii) *Destination-bottleneck requirement* - the maximum amount of data that can be simultaneously received by a destination node; (iv) *Interface-bottleneck requirement* - the maximum amount of data that an interface can simultaneously transmit or receive. We also found that each of the four requirements dominates the other three requirements in terms of the throughput of the network under different conditions on C_A and H. Specifically, C_A can be partitioned into 3 cases: (1) the case when $C_A = O(F_1)$, (2) the case when $C_A = \Omega(F_1)$ and $C_A = O(F_2)$, and (3) the case when $C_A = \Omega(F_2)$, where $F_1 = \log n$ and $F_2 = n(\frac{\log\log(H^2 \log n)}{\log(H^2 \log n)})^2$.

Under each of the above cases, H can be partitioned into two sub-cases. Under the first case, H is partitioned into 2 sub-cases, namely Sub-case 1 (when $H = o(G_1)$) and Sub-case 2 (when $H = \Omega(G_1)$), where $G_1 = n^{\frac{1}{3}} / \log^{\frac{2}{3}} n$. Under the second case, H is partitioned into 2 sub-cases, namely Sub-case 3 (when $H = o(G_2)$) and Sub-case 4 (when $H = \Omega(G_2)$), where $G_2 = n^{\frac{1}{3}} C_A^{\frac{1}{6}} / \log^{\frac{1}{2}} n$. Under the third case, H is partitioned into 2 sub-cases, namely Sub-case 5 (when $H = o(G_3)$) and Sub-case 6 (when $H = \Omega(G_3)$), where $G_3 = n^{\frac{1}{2}} / \log^{\frac{1}{2}} n$. Fig. 2 shows all possible sub-cases. Specifically, each requirement dominates the other at least one sub-case under different conditions as follows.

- *Connectivity Condition*: corresponding to Sub-case 2 in which Connectivity requirement dominates.
- *Interference Condition*: corresponding to Sub-case 4 in which *Interference requirement* dominates.
- *Destination-bottleneck Condition*: corresponding to Sub-case 6 in which *Destination-bottleneck requirement* dominates.
- *Interface-bottleneck Condition*: corresponding to Sub-case 1, Sub-case 3, or Sub-case 5, in which *Interface-bottleneck requirement* dominates.

II. Formulation and Models

A. Interference model

We consider the *interference* model [1]–[6]. When node X_1 transmits to node X_2 over a particular channel, the transmission is successfully completed by node X_2 if no node within the transmission range of X_2 transmits over the same channel. Therefore, for any other node X_3 simultaneously transmitting over the same channel, and any guard zone $\Delta > 0$, the following condition holds.

$$\mathrm{dist}(X_3, X_2) \geq (1+\Delta)\mathrm{dist}(X_1, X_2)$$

where $\mathrm{dist}(X_1, X_2)$ denotes the distance between two nodes X_1 and X_2. The interference model applies for both ad hoc communications and infrastructure communications. Since ad hoc communications and infrastructure communications are separated by different channels (i.e., C_A and C_I), the interference only occurs either between two ad hoc communications or between two infrastructure communications.

B. Definitions of Throughput Capacity and Delay

Definition 1: Feasible per-node throughput. For an *MC-IS* network, a throughput of λ (in bits/sec) is *feasible* if by ad hoc communications or infrastructure communications, there exists a *spatial and temporal scheme*, within which each node can send or receive λ bits/sec on average.

Definition 2: Per-node throughput capacity with the throughput of λ is of order $\Theta(g(n))$ bits/sec if there are deterministic constants $h > 0$ and $h' < +\infty$ such that

$$\begin{array}{ll} \lim_{n\to\infty} & P(\lambda = hg(n) \text{ is feasible}) = 1 \text{ and} \\ \lim_{n\to\infty} \inf & P(\lambda = h'g(n) \text{ is feasible}) < 1 \end{array}$$

Besides, we use T and T_A to denote the *feasible aggregate throughput* and the *feasible* aggregate throughput contributed by *ad hoc* communications, respectively.

The *delay of a packet* D is defined as the time that it takes for the packet to reach its destination after it leaves the source [7]. Averaging the delay of all the packets transmitted in the whole network, we obtain the *average delay* of a network.

III. Upper Bounds on Network Capacity Contributed by Ad Hoc Communications

We first derive the upper bounds on the per-node throughput capacity under Connectivity Condition.

Proposition 1: When Connectivity requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O(\frac{nW_A}{H^3 C_A \log^2 n})$.

Proof. We first calculate the expectation of the number of hops under the H-max-hop routing scheme, which is denoted by $\overline{h}$

$$\begin{aligned} \overline{h} = E(h) &= 1\cdot P(h=1) + 2\cdot P(h=2) + \ldots \\ &\quad +H\cdot P(h=H) \\ &= 1\cdot\frac{\pi r^2(n)}{\pi H^2 r^2(n)} + 2\cdot\frac{3\pi r^2(n)}{\pi H^2 r^2(n)} + \ldots \\ &\quad +H\cdot\frac{(H^2-(H-1)^2)\pi r^2(n)}{\pi H^2 r^2(n)} \\ &= \frac{4H^3+3H^2-H}{6H^2} \end{aligned} \tag{1}$$

where $P(h=i)$ $(i = 1, 2, \ldots, H)$ is the probability that a packet traverses $h = i$ hops.

From Eq. (1), we have $\overline{h} \sim H$.

We then calculate the probability that a node uses the ad hoc mode to transmit, denoted by $P(AH)$, which is the probability that the destination node is located within H hops away from the source node. Thus, we have

$$P(AH) = \pi H^2 r^2(n) \tag{2}$$

Since each source generates λ_a bits per second and there are totally n sources, the total number of bits per second served by the whole network on a particular channel is required to be at least $n\cdot P(AH)\cdot\overline{h}\cdot\lambda_a$, which is bounded by $N_{max}\cdot\frac{W_A}{C_A}$, where N_{max} is the maximum number of simultaneous transmissions on any particular channel, which is upper bounded by $N_{max} \leq \frac{k_1}{\Delta^2(r(n))^2}$ ($k_1 > 0$ is a constant, independent of n) [1]. Then, we have $n\cdot P(AH)\cdot\overline{h}\cdot\lambda_a \leq N_{max}\cdot\frac{W_A}{C_A}$.

Combining the above results yields:

$$\lambda_a \leq \frac{k_1}{\Delta^2 r^2(n)}\cdot\frac{W_A}{n\pi H^3 r^2(n) C_A} \leq \frac{k_2 W_A}{nH^3 r^2(n) C_A}$$

where k_2 is a constant.

Besides, to guarantee that the network is connected with high probability (*w.h.p.*)[1], we require $r(n) > \sqrt{\log n/\pi n}$ [1]. Thus, we have $\lambda_a \leq \frac{k_3 nW_A}{H^3\log^2 nC_A}$, where k_3 is a constant. □

We then derive the upper bounds on the per-node throughput capacity under Interference Condition.

Proposition 2: When Interference requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O(\frac{nW_A}{C_A^{\frac{1}{2}} H^3 \log^{\frac{3}{2}} n})$.

Proof. When Interference Condition is satisfied, the per-node throughput is limited by the interference requirement [1]. Thus, we can use the theorem derived under arbitrary networks [1]. Similarly, we assume that all nodes are synchronized. Let the average distance between a source and a destination be $\overline{l}$, which is roughly bounded by $\overline{h}\cdot r(n)$.

In the network with n nodes and under the H-max-hop routing scheme, there are at most $n\cdot P(\text{AH})$, where $P(AH)$ is the probability that a node transmits in ad hoc mode and can be calculated by Eq. (2). Within any time period, we consider a bit b, $1 \leq b \leq \lambda nP(AH)$ We assume that bit b traverses $h(b)$

[1] We say that an event e happens with a high probability if $P(e) \to 1$ when $n \to \infty$.

hops on the path from the source to the destination, where the h-th hop traverses a distance of $r(b,h)$. It is obvious that the distance traversed by a bit from the source to the destination is no less than the length of the line jointing the source and the destination. Thus, after summarizing the traversing distance of all bits, we have

$$\lambda_a \cdot n\bar{l} \cdot P(AH) \leq \sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h)$$

Let T_h be the total number of hops traversed by all bits in a second and we have $T_h = \sum_{b=1}^{n\lambda_a P(AH)} h(b)$. Since each node has one interface which can transmit at most $\frac{W_A}{C_A}$, the total number of bits that can be transmitted by all nodes over all interfaces are at most $\frac{W_A n}{2C_A}$, i.e.,

$$T_h \leq \frac{W_A n}{2C_A} \tag{3}$$

On the other hand, under the interference model, we have the following in-equation from [1]

$$\text{dist}(X_1 - X_2) \geq \frac{\Delta}{2}(\text{dist}(X_3 - X_4) + \text{dist}(X_1 - X_2))$$

where X_1 and X_3 denote the transmitters and X_2 and X_4 denote the receivers. This in-equation implies that each hop consumes a disk of radiums $\frac{\Delta}{2}$ times the length of the hop.

Therefore, we have

$$\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} \frac{\pi\Delta^2}{4}(r(b,h))^2 \leq W_A$$

This in-equation can be rewritten as

$$\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} \frac{1}{T_h}(r(b,h))^2 \leq \frac{4W_A}{\pi\Delta^2 T_h} \tag{4}$$

Since the left hand side of this in-equation is convex, we have

$$(\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} \frac{1}{T_h} r(b,h))^2 \leq \sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} \frac{1}{T_h}(r(b,h))^2 \tag{5}$$

Joining (4)(5), we have

$$\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h) \leq \sqrt{\frac{4W_A T_h}{\pi\Delta^2}}$$

From (3), we have

$$\sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h) \leq W_A\sqrt{\frac{2n}{\pi\Delta^2 C_A}}$$

Besides, since $\lambda_a \cdot n\bar{l} \cdot P(AH) \leq \sum_{b=1}^{n\lambda_a P(AH)} \sum_{h=1}^{h(b)} r(b,h)$, we have

$$\lambda_a \leq \frac{W_A\sqrt{\frac{2n}{\pi\Delta^2 C_A}}}{n\bar{l} \cdot P(AH)} = \frac{W_A\sqrt{\frac{2n}{\pi\Delta^2 C_A}}}{n\bar{h}r(n)\pi H^2(r(n))^2} \leq \frac{W_A\sqrt{\frac{2}{\pi\Delta^2 nC_A}}}{\pi H^3(r(n))^3}$$

Since $r(n) > \sqrt{\frac{\log n}{\pi n}}$, we have

$$\lambda_a \leq \frac{k_4 n W_A}{C_A^{\frac{1}{2}} H^3 \log^{\frac{3}{2}} n}$$

□

Before deriving the upper bounds on the throughput capacity under the destination-bottleneck condition, we need to bound the number of flows towards a node under the H-max-hop routing scheme. Specifically, we have the following result.

Lemma 1: The maximum number of flows towards a node under the H-max-hop routing scheme is $D_H(n) = \Theta(\frac{\log(H^2 \log n)}{\log\log(H^2 \log n)})$ with high probability (*w.h.p.*).

Proof. As shown in [6], the total number of source nodes transmitting in ad hoc mode under the H-max-hop routing scheme is $\Theta(H^2 \log n)$ *w.h.p.*. Besides, it is proved in [8] that the maximum number of flows towards any given node in a random ad hoc network with n nodes is upper bounded by $\Theta(\frac{\log n}{\log\log n})$ *w.h.p.*. Combining the two results leads to the above result. □

We then obtain the upper bounds on the per-node throughput capacity under Destination-bottleneck Condition.

Proposition 3: When Destination-bottleneck requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O(n^{\frac{3}{2}} \log\log(H^2 \log n) W_A / (C_A H^3 \log^{\frac{3}{2}} n \cdot \log(H^2 \log n)))$.

Proof. Each node has one interface that can support at most $\frac{W_A}{C_A}$. Since each node has at most $D_H(n)$ flows under the H-max-hop routing scheme, the data rate of the minimum rate flow is at most $\frac{W_A}{C_A D_H(n)}$, where $D_H(n)$ is bounded by $\Theta(\frac{\log(H^2 \log n)}{\log\log(H^2 \log n)})$ by Lemma 1. After calculating all the data rates at each node times with the traversing distance, we have $n \cdot P(AH) \cdot \lambda_A \cdot \bar{h} \cdot r(n) \leq \frac{W_A n}{C_A D_H(n)} \cdot 1$, where $P(AH)$ and $\bar{h}$ are defined in the proof of Proposition 1. We then have $\lambda_A \leq \frac{W_A}{C_A D_H(n) P(AH) \bar{h} r(n)} \leq \frac{W_A}{C_A \pi H^3 r^3(n) \cdot \log(H^2 \log n)/\log\log(H^2 \log n)}$ since $\bar{h} \sim H$ and $P(AH) = \pi H^2 r^2(n)$ as shown in the proof of Proposition 1. Since $r(n) = \Theta(\sqrt{\log n/n})$ [1], we have $\lambda_A \leq W_A n^{\frac{3}{2}} \cdot \log\log(H^2 \log n)/(C_A H^3 \log^{\frac{3}{2}n} \cdot \log(H^2 \log n))$ □

Finally, we prove the upper bounds on the per-node throughput capacity under Interface-bottleneck Condition.

Proposition 4: When Interface-bottleneck requirement dominates, the per-node throughput capacity contributed by ad hoc communications is $\lambda_a = O(\frac{W_A}{C_A})$.

Proof. In an *MC-IS* network, each node is equipped with only one NIC supporting at most $\frac{W_A}{C_A}$ data rate. Thus, λ_a is also upper bounded by $\frac{W_A}{C_A}$ for any network settings. □

IV. CONSTRUCTIVE LOWER BOUNDS ON NETWORK CAPACITY CONTRIBUTED BY AD HOC COMMUNICATIONS

We first divide the plane into a number of equal-sized cells. The size of each cell is properly chosen so that each cell has $\Theta(na(n))$ nodes, where $a(n)$ is the area of a cell. We then design a routing scheme to assign the number of flows at each node evenly. Finally, we design a *Time Division Multiple Access* (TDMA) scheme to schedule the traffic at each node.

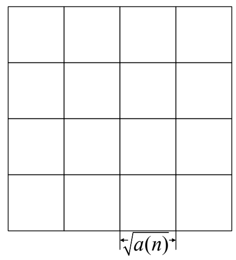

Fig. 3. Plane divided into a number of cells and each with area $a(n)$.

A. Cell Construction

We divide the plane into $1/a(n)$ equal-sized cells and each cell is a square with area of $a(n)$, as shown in Fig. 3. The cell size of $a(n)$ must be carefully chosen to fulfill the three requirements, i.e., Connectivity requirement, Interference requirement and Destination-bottleneck requirement. We set $a(n) = \min(\max(\frac{100\sqrt{C_A}\log n}{n}, \log^{\frac{3}{2}} n/C_A^{\frac{1}{2}}n), \frac{\log^{\frac{3}{2}} n\cdot\log(H^2\log n)}{n^{\frac{3}{2}}\cdot\log\log(H^2\log n)})$ similar to [4]. Note that Interface-bottleneck requirement is independent of the size of a cell.

The maximum number of nodes in a cell is bounded by the following lemma.

Lemma 2: [4] If $a(n) > \frac{50\log n}{n}$, then each cell has $\Theta(n(a(n))$ nodes *w.h.p.*.

We next check whether all the above values of $a(n)$ are properly chosen such that each cell has $\Theta(n(a(n))$ nodes *w.h.p.* when n is large enough (i.e., Lemma 2 is satisfied). It is obvious that $\frac{100\sqrt{C_A}\log n}{n} > \frac{50\log n}{n}$ and $\log^{\frac{3}{2}} n/(C_A^{\frac{1}{2}}n) > \frac{50\log n}{n}$ (as we only consider C_A in Connectivity Condition and Interference Condition). Besides, $\frac{\log^{\frac{3}{2}} n\cdot\log(H^2\log n)}{n^{\frac{3}{2}}\cdot\log\log(H^2\log n)}$ is also greater than $\frac{50\log n}{n}$ with large n since $\frac{\log(H^2\log n)}{\log\log(H^2\log n)} > 1$ and $\log^{\frac{3}{2}} n/n^{\frac{3}{2}} > \frac{50\log n}{n}$ when n is large enough.

It is also proved in [7], [9] that the number of interfering cells around a cell is bounded by a constant k_5, which is independent of n.

B. Routing Scheme

To assign the flows at each node evenly, we design a routing scheme consists of two steps: (1) Assigning sources and destinations and (2) Assigning the remaining flows in a balanced way.

In Step (1), each node is the originator of a flow and each node is the destination of at most $D_H(n)$ flows, where $D_H(n)$ is defined in Lemma 1. Thus, after Step (1), there are at most $1 + D_H(n)$ flows.

We denote the straight line connecting a source S to its destination D as an S-D lines. In Step (2), we need to calculate the number of S-D lines (flows) passing through a cell so that we can assign them to each node evenly. Specifically, we have the following result.

Lemma 3: The number of S-D lines passing through a cell is bounded by $O(nH^3(a(n))^2)$.

Proof. We present a proof of the bound in [9]. □

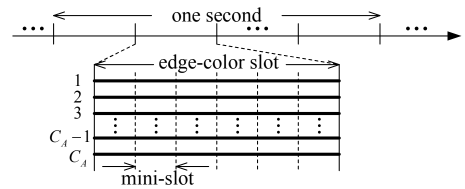

Fig. 4. TDMA transmission schedule

As shown in Lemma 2, there are $\Theta(n \cdot a(n))$ nodes in each cell. Therefore, Step (2) will assign to any node at most $O(\frac{nH^3(a(n))^2}{n\cdot a(n)}) = O(H^3a(n))$ flows. Summarizing Step (1) and Step (2), there are at most $f(n) = O(1 + H^3a(n) + D_H(n))$ flows at each node. On the other hand, $H^3a(n)$ dominates $f(n)$ since $H > 1$ and $a(n)$ is asymptotically larger than $D_H(n)$ when n is large enough. Thus, we have $f(n) = O(H^3a(n))$.

C. Scheduling Transmissions

We next design a scheduling scheme to transmit the traffic flows assigned in a *routing scheme*. Any transmissions in this network must satisfy the two additional constraints simultaneously: 1) each interface only allows one transmission/reception at the same time, and 2) any two transmissions on any channel should not interfere with each other.

We propose a TDMA scheme to schedule transmissions that satisfy the above two constraints. Fig. 4 depicts a schedule of transmissions on the network. In this scheme, one second is divided into a number of *edge-color* slots and at most one transmission/reception is scheduled at every node during each edge-color slot. So, the first constraint is satisfied. Each edge-color slot can be further split into smaller *mini-slots*. In each mini-slot, each transmission satisfies the above two constraints.

Then, we describe the two time slots as follows.

(i) *Edge-color slot*: First, we construct a routing graph in which vertices are the nodes in the network and an edge denotes transmission/reception of a node. In this construction, one hop along a flow is associated with one edge in the routing graph. In the routing graph, each vertex is assigned with $f(n) = O(H^3a(n))$ edges, which can be edge-colored with at most $O(H^3a(n))$ colors [4], [10]. We then divide one second into $O(H^3a(n))$ edge-color slots, each of which has a length of $\Omega(\frac{1}{H^3a(n)})$ seconds and is stained with a unique edge-color. Since all edges connecting to a vertex use different colors, each node has at most one transmission/reception scheduled in any edge-color time slot.

(ii) *Mini-slot*: We further divide each edge-color slot into mini-slots. Then, we build a schedule that assigns a transmission to a node in a mini-slot within an edge-color slot over a channel. We construct an *interference graph* in which each vertex is a node in the network and each edge denotes the interference between two nodes. We then show as follows that the interference graph can be vertex-colored with $k_7(na(n))$ colors, where k_7 is a constant defined in [4].

Lemma 4: The interference graph can be vertex-colored with at most $O(na(n))$ colors.

Proof. We present the detailed proof in [9]. $\square$

We need to schedule the interfering nodes either on different channels, or at different mini-slots on the same channel since two nodes assigned the same vertex-color do not interfere with each other, while two nodes stained with different colors may interfere with each other. We divide each edge-color slot into $\lceil k_7 na(n)/C_A \rceil$ mini-slots on every channel, and assign the mini-slots on each channel from 1 to $\lceil k_7 na(n)/C_A \rceil$. A node assigned with a color s, $1 \leq s \leq k_7 na(n)$, is allowed to transmit in mini-slot $\lceil s/C_A \rceil$ on channel $(s \bmod C_A) + 1$.

We next have the constructive lower bounds of the capacity.

Proposition 5: The achievable per-node throughput capacity λ_a contributed by ad hoc communications is as follows.

1) When Connectivity requirement dominates, λ_a is $\Omega(\frac{nW_A}{H^3 C_A \log^2 n})$ bits/sec;
2) When Interference requirement dominates, λ_a is $\Omega(\frac{nW_A}{H^3 C_A^{\frac{1}{2}} \log^{\frac{3}{2}} n})$ bits/sec;
3) When Destination-bottleneck requirement dominates, λ_a is $\Omega(\frac{n^{\frac{3}{2}} \log\log(H^2 \log n) W_A}{C_A H^3 \log^{\frac{3}{2}} n \cdot \log(H^2 \log n)})$ bits/sec;
4) When Interface-bottleneck requirement dominates, λ_a is $\Omega(\frac{W_A}{C_A})$.

Proof. Since each edge-color slot with a length of $\Omega(\frac{1}{H^3 a(n)})$ seconds is divided into $\left\lceil \frac{k_7 na(n)}{C_A} \right\rceil$ mini-slots over every channel, each mini-slot has a length of $\Omega((\frac{1}{H^3 a(n)}) / \left\lceil \frac{k_7 na(n)}{C_A} \right\rceil)$ seconds. Since each channel can transmit at the rate of $\frac{W_A}{C_A}$ bits/sec, in each mini-slot, $\lambda_a = \Omega(\frac{W_A}{(C_A H^3 a(n) \cdot \left\lceil \frac{k_7 na(n)}{C_A} \right\rceil})$ bits can be transported. Since $\left\lceil \frac{k_7 na(n)}{C_A} \right\rceil \leq \frac{k_7 na(n)}{C_A} + 1$, we have, $\lambda_a = \Omega(\frac{W_A}{k_7 H^3 a^2(n) n + H^3 a(n) C_A})$ bits/sec. Thus, $\lambda_a = \Omega(MIN_O(\frac{W_A}{H^3 a^2(n) n}, \frac{W_A}{H^3 a(n) C_A}))$ bits/sec (where $MIN_O(f(n), g(n))$ is equal to $f(n)$ if $f(n) = O(g(n))$; otherwise it is equal to $g(n)$).

Recall that $a(n)$ is set to $\min(\max(\frac{100 C_A^{\frac{1}{2}} \log n}{n}, \frac{\log^{\frac{3}{2}} n}{C_A^{\frac{1}{2}} n}), \frac{\log^{\frac{3}{2}} n \cdot \log(H^2 \log n)}{n^{\frac{3}{2}} \cdot \log\log(H^2 \log n)})$. Substituting the three values to λ_a, we have the results 1), 2) and 3). Besides, each interface can transmit or receive at the rate of $\frac{W_A}{C_A}$ bits/sec. Thus, $\lambda_a = \Omega(\frac{W_A}{C_A})$, which is the result 4). $\square$

D. Summary

It is shown in [6] that the total traffic of ad hoc communications is $n\pi H^2 r^2(n) \lambda_A$. Combining Propositions 1, 2, 3, 4 and 5 leads to the following theorem.

Theorem 1: The aggregate throughput capacity of the network contributed by ad hoc communications is

1) When Connectivity requirement dominates, T_A is $\Theta(\frac{nW_A}{HC_A \log n})$ bits/sec.
2) When Interference requirement dominates, T_A is $\Theta(\frac{nW_A}{C_A^{\frac{1}{2}} H \log^{\frac{1}{2}} n})$ bits/sec.
3) When Destination-bottleneck requirement dominates, T_A is $\Theta(\frac{n^{\frac{3}{2}} \log\log(H^2 \log n) W_A}{C_A H \log^{\frac{1}{2}} n \cdot \log(H^2 \log n)})$ bits/sec.
4) When Interface-bottleneck requirement dominates, T_A is $\Theta(H^2 \log n \cdot \frac{W_A}{C_A})$ bits/sec.

We then derive the average delay of an *MC-IS* network and have the following result.

Theorem 2: Under the H-max-hop ad hoc routing strategy, if the packets are transmitted in the ad hoc mode and along a route which approximates the straight line connecting the source and the destination, the average delay is $\Theta(H)$.

Proof. The average delay of the packets transmitted in the ad hoc mode under the H-max-hop routing strategy in an *SC-IS* network is bounded by $\Theta(H)$ [6], which also holds for an *MC-IS* network since both an *SC-IS* network and an *MC-IS* network have the same routing strategy. $\square$

V. Conclusion

In this paper, we propose a novel multi-channel wireless network with infrastructure (named an *MC-IS* network), which consists of common nodes, each with a single interface, and infrastructure nodes, each with multiple interfaces. We derive the upper bounds and lower bounds on the capacity of an *MC-IS* network contributed by ad hoc communications, where the upper bounds are proved to be tight. We also prove that the average delay contributed by ad hoc communications is bounded by H, which is the maximum number of hops in H-max routing scheme.

There are some interesting questions in this new type of networks: (1) what are the upper bounds on the capacity of an *MC-IS* network contributed by infrastructure communications? (2) are the upper bounds also tight? (3) what is the average delay of an *MC-IS* network with considering both ad hoc communications and infrastructure communications? To solve the above questions would be one of our future works.

VI. Acknowledgment

The work described in this paper was supported by Macao Science and Technology Development Fund under Grant No. 036/2011/A and Grant No. 081/2012/A3. The authors would like to thank Gordon G.-D. Han for his constructive comments.

References

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388 – 404, 2000.

[2] B. Liu, Z. Liu, and D. Towsley, "On the Capacity of Hybrid Wireless Networks," in *Proc. of IEEE INFOCOM*, 2003.

[3] U. C. Kozat and L. Tassiulas, "Throughput capacity of random ad hoc networks with infrastructure support," in *Proc. of ACM MobiCom*, 2003.

[4] P. Kyasanur and N. H. Vaidya, "Capacity of multichannel wireless networks: Impact of number of channels and interfaces," in *Proc. of ACM MobiCom*, 2005.

[5] A. Zemlianov and G. D. Veciana, "Capacity of ad hoc wireless networks with infrastructure support," *IEEE JSAC*, vol. 23, p. 657, 2005.

[6] P. Li, C. Zhang, and Y. Fang, "Capacity and delay of hybrid wireless broadband access networks," *IEEE JSAC*, vol. 27, pp. 117–125, 2009.

[7] A. Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," in *INFOCOM*, 2004.

[8] M. Raab and A. Steger, "Balls into bins - A simple and tight analysis," *Lecture Notes in Computer Science*, vol. 1518, pp. 159–170, 1998.

[9] H.-N. Dai, R. C.-W. Wong, and Q. Zhao, "Multi-channel wireless networks with infrastructure support: Capacity and delay," Tech. Rep., Feb. 2014. [Online]. Available: http://arxiv.org/pdf/1310.1530v2.pdf

[10] D. B. West, *Introduction to graph theory*, 2nd ed. Upper Saddle River, N.J.: Prentice Hall PTR, 2001.